PROPAGATION OF A ONE-DIMENSIONAL PLASTIC WAVE IN A MEDIUM WITH LINEAR AND DISCONTINUOUS UNLOADINGS

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The propagation of plane and spherical waves in a nonlinearly compressed medium with linear and discontinuous unloadings when acted upon by intense loads is considered. The solutions of the problem are constructed by an inverse method [1] assuming that the medium at the front of the shock wave is instantaneously loaded nonlinearly, while behind the shock wavefront in the perturbed region irreversible unloading of the medium occurs. The propagation and reflection of elastoplastic waves in a rod of finite length for the Prandtl arrangement with discontinuous unloading has been solved by the method of characteristics in [2].

Unlike [2] in this paper we solve the one-dimensional nonstationary problems of a plane and spherical layer analytically using an inverse method, and we consider the propagation of a nonlinear loading–unloading shock wave. It should be noted that this paper is a continuation of [1] for a medium with discontinuous unloading. In the case of linear unloading of the medium, the finiteness of the time interval during which the load applied to the boundary layer acts is taken into account and solutions of the problem are found in regions outside the limits in which it acts. The inverse method consists in determining the wave field in a layer of ground and the profile of the load applied to its boundary from the products of an explosion with a specified law of motion of the shock wave. The ground when acted upon by intense forces, as in [3], is taken to be a nonlinearly compressed ideal medium. A similar approach was used previously in [4] to study the mechanical action of an underground explosion. For a specific structure of the medium the results of the calculations are presented in the form of graphs of the pressure, and of the velocity of the medium at the boundary of the layer, on the shock-wave front, and in the perturbed region as a function of time. A detailed analysis is carried out of the kinematic parameters of the medium for the case of linear unloading, and a comparison is made with acoustics. The effect of the nonlinear properties of the medium on the distribution of the dynamic characteristics of shock-wave processes is investigated. The calculations were carried out for the case when the velocity of the shock-wave front is specified in the form of a linearly decreasing function of time, and when solving the problem the corresponding load profile is determined. The surface of the pressure isobars is constructed.

1. <u>The Propagation of Plane and Spherical Waves in a Nonlinearly Compressed Medium with Linear Unloading.</u> Suppose a monotonically decreasing load $p_0(t)$ is applied to the boundary of the layer $r = R_0$. The equations in the unloading region, the relations on the front $r = R(t)$, and the boundary condition (the initial conditions are zero) have the form [1]

$$\frac{\partial u}{\partial t}+\frac{1}{\rho_0}\frac{\partial p}{\partial r}=0,\quad \frac{\partial \rho}{\partial t}+\rho_0\left(\frac{\partial u}{\partial r}+\nu\frac{u}{r}\right)=0, \tag{1.1}$$

$$p\ (r,\ t)=p^*+\mathrm{E}(\varepsilon-\varepsilon^*),\quad \varepsilon=1-\frac{\rho_0}{\rho},\quad \mathrm{E}=c_p^2\rho_0;$$

$$u^*(t)=\varepsilon^*\dot{R},\ p^*=\rho^*\varepsilon^*\dot{R}^2, \tag{1.2}$$

$$p^*(t)=\alpha_1\varepsilon^*+\alpha_2\varepsilon^{*2}\ (\dot{R}=dR/dt)\ \text{for}\quad r=R(t);$$

$$p(r,\ t)=p_0(t)\ \text{for}\quad r=R_0, \tag{1.3}$$

where u is the mass velocity, ρ is the density, p is the pressure, ε is the volume deformation, $\nu = 0, 2$ relate to the plane and spherical layer respectively, and the parameters of the medium which relate to the front are denoted by the asterisk. If we specify the velocity of the front in the form of a decreasing function of time, all the parameters of the medium when $r = R(t)$ will be known, and relations (1.2) will be the boundary condition for (1.1).

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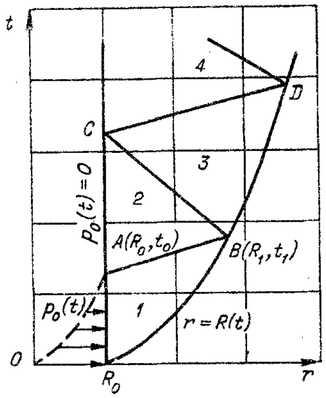

Fig. 1

In this case we obtain the following equation for a plane one-dimensional wave ($\nu = 0$) from (1.1):

$$\frac{\partial^2 u}{\partial t^2} - c_p^2 \frac{\partial^2 u}{\partial r^2} = 0, \tag{1.4}$$

which, taking (1.2) into account, has a solution in the form

$$u(r, t) = u^*(0) - \frac{1}{2c_p} \sum_{i=1}^{2} (-1)^{i-1} \int_{R_0}^{r-(-1)^{i-1}c_p t} \ddot{R}[F(z_i)] \left\{ \Delta_1 [F(z_i)] + \frac{\frac{\rho_0}{\alpha_2}\left[2\dot{R}(F(z_i)) + (-1)^{i-1} c_p\right] \dot{R}[F(z_i)]}{\Delta_2 [F(z_i)]} \right\} dz_i, \tag{1.5}$$

where $\Delta_1(t) = \dfrac{\left(1 - \dfrac{\alpha_1}{\alpha_2}\right)}{2} - \Delta_2(t)$; $\Delta_2(t) = \sqrt{\dfrac{\left(1 - \dfrac{\alpha_1}{\alpha_2}\right)^2}{4} - \dfrac{\rho_0 \dot{R}^2(t) - \alpha_1}{\alpha_2}}$.

Substituting (1.5) into the first of Eqs. (1.1) and integrating with respect to r from $r = R_0$ to $r = R(t)$, we obtain the following equation for determining the load $p_0(t)$:

$$p_0(t) = p^*(t) + \frac{\rho_0}{2} \int_{R_0}^{R(t)} \sum_{i=1}^{2} \ddot{R}[F(z_i)] \left\{ \Delta_1 [F(z_i)] + \frac{\frac{\rho_0}{\alpha_2}\left[2\dot{R}(F(z_i)) + (-1)^{i-1} c_p\right] \dot{R}(F(z_i))}{\Delta_2 [F(z_i)]} \right\} dr, \tag{1.6}$$

where $z_{1,2} = r \mp c_p t$, and $F(z_{1,2})$ is the root of the equation $R(t) \mp c_p t = z_{1,2}$ for t. Note that Eq. (1.6) is more accurate than the one in [1] and holds so long as $p_0(t) \geqslant 0$. We will now solve the corresponding boundary value problems. The region considered is divided into $n = 1, 2, 3, \ldots$ regions each of which for $n \geqslant 2$ is bounded by the characteristics AB, BC, CD, etc. (Fig. 1), of the positive and negative directions, the boundary layer, or the part of the front $r = R(t)$.

We will give the solution of the problem for region 2. From (1.1) for $\nu = 0$ we obtain the equation

$$\frac{\partial^2 p}{\partial t^2} - c_p^2 \frac{\partial^2 p}{\partial r^2} = 0,$$

for the solution of which we have the conditions

$$p(r, t) = p_1(t) \quad \text{for} \quad r - c_p t = R_0 - c_p t_0,$$
$$p(r, t) = 0 \quad \text{for} \quad r = R_0,\ t \geqslant t_0.$$

Then, using d'Alembert's formula we obtain

$$p(r, t) = -p_1\left[\frac{(R_0 + c_p t_0) - (r - c_p t)}{2c_p}\right] + p_1\left[\frac{(r + c_p t) - (R_0 - c_p t_0)}{2c_p}\right].$$

Integrating the first equation of (1.1) with respect to t from $t = t^*(r)$ to t, we have

$$u(r,t) = u_1\left[r, \frac{r-(R_0-c_p t_0)}{c_p}\right] - \frac{1}{\rho_0 c_p}\left\{p_1\left[\frac{(R_0+c_p t_0)-(r-c_p t)}{2c_p}\right] + p_1\left[\frac{(r+c_p t)-(R_0-c_p t_0)}{2c_p}\right] - p_1(t) - p_1(t_0)\right\},$$

where $p_1(t)$ and $u_1(t)$ are the pressure and velocity of the medium along the characteristic AB, determined from the solution of the problem in region 1. The solution of Eq. (1.4) in the region 3 can be represented in the form

$$u(r,t) = f_5(r - c_p t) + f_6(r + c_p t). \tag{1.7}$$

To find the functions f_5 and f_6 the problem has a boundary condition on BC and relations on the front $r = R(t)$. However, as calculations show [5], the front of a two-dimensional stationary plastic wave, depending on the depth of the half-plane, is only slightly changed. The deformation of the front compared with the initial shape is approximately 15-20% and is even less for greater depths. In addition, the line BD has a finite length. Hence, to a first approximation the relations on the discontinuity are satisfied by the initial shape of the front, corresponding to the point B (R_1, t_1). We then have

$$u(r,t) = u_2(t) \text{ when } r + c_p t = R_1 + c_p t_1; \tag{1.8}$$

$$\left.\begin{aligned} u(r,t) &= \dot{R}(t)\Delta_1(t) \\ \frac{\partial u}{\partial r} &= -\frac{\frac{\rho_0}{\alpha_2}\dot{R}(t)\ddot{R}(t)}{\Delta_2(t)} \end{aligned}\right\} \text{when } r - R_1 \approx a_1(t - t_1), \tag{1.9}$$

where $a_1 = dR/dt$ when $r = R_1$, and $t = t_1$, and u_2 is the velocity of the medium at BC, found from the solution in region 2. Substituting (1.7) into (1.8) we obtain

$$u(r,t) = u_2\left[\frac{r_1 + c_p t_1 - (r - c_p t)}{2c_p}\right] + f_6(r + c_p t) - f_6(r_1 + c_p t_1). \tag{1.10}$$

System (1.9), taking (1.10) into account, enables one to obtain the following system of two equations for $f_6(t)$ and $\dot{R}(t)$ (in region 3 unlike region 1 $\dot{R}$ is the desired parameter):

$$u_2\left[\frac{\left(1+\frac{a_1}{c_p}\right)t_1 + \left(1-\frac{a_1}{c_p}\right)t}{2}\right] - u_2(t_1) = \frac{\left(1-\frac{\alpha_1}{\alpha_2}\right)}{2}(\dot{R}(t) - a_1) - (\dot{R}(t) + c_p + a_1)\Delta_2[\dot{R}(t)] + (c_p + 2a_1)\Delta_2(a_1); \tag{1.11}$$

$$u_2\left[\frac{\left(1+\frac{a_1}{c_p}\right)t_1 + \left(1-\frac{a_1}{c_p}\right)t}{2}\right] + f_6[(R_1 - a_1 t_1) + (a_1 + c_p)t] - f_6(R_1 + c_p t_1) = \dot{R}(t)\Delta_1[\dot{R}(t)]. \tag{1.12}$$

Equation (1.11) for $\dot{R}(t)$ is easily solved by a graphical-analytical method. After finding $\dot{R}(t)$, using (1.11), we find $f_6(t)$ from (1.12), and then the mass velocity from (1.10). Further, by integrating the equation of motion of system (1.1) with respect to r from $r = -c_p t + (R_1 + c_p t_1) = R_2(t)$ to r, we obtain

$$p(r,t) = p_2(t) - \rho_0 \int_{R_2(t)}^{r} \frac{\partial u}{\partial t}\,dr,$$

where $p_2(t)$ is the pressure at BC, calculated from the solution in region 2.

The solutions of the problem for the following regions (see Fig. 1) are obtained in the same way. In the case of a spherical wave ($\nu = 2$) from (1.1) we have

$$\frac{\partial^2 u}{\partial t^2} - c_p^2\frac{\partial^2 u}{\partial r^2} - 2\frac{c_p^2}{r}\left(\frac{\partial u}{\partial r} - \frac{u}{r}\right) = 0,$$

which, taking (1.2) into account for given $\dot{R}(t)$ gives a solution of the form

$$u(r,t)=\frac{1}{r}\left\{\int\limits_{\dot{R}_0}^{r-c_pt}d\xi_2\int\limits_{\dot{R}_0}^{\xi_2}\Phi(\xi_1)\,d\xi_1-\int\limits_{\dot{R}_0}^{r+c_pt}d\xi_2\int\limits_{\dot{R}_0}^{R[F(\xi_2)]-c_pF(\xi_2)}\Phi(\xi_1)\,d\xi_1-\right. \tag{1.13}$$

$$\left.-\frac{\rho_0}{\alpha_2}\int\limits_{\dot{R}_0}^{r+c_pt}\frac{\dot{R}\,[F(\xi_2)]\,\ddot{R}\,[F(\xi_2)]\,R\,[F(\xi_2)]}{\Delta_2\,[F(\xi_2)]}\,d\xi_2\right\}-\frac{1}{r^2}\left\{\int\limits_{\dot{R}_0}^{r-c_pt}d\xi_3\int\limits_{\dot{R}_0}^{\xi_3}d\xi_2\int\limits_{\dot{R}_0}^{\xi_2}\Phi(\xi_1)\,d\xi_1-\int\limits_{\dot{R}_0}^{r+c_pt}d\xi_3\int\limits_{\dot{R}_0}^{\xi_3}d\xi_2\int\limits_{\dot{R}_0}^{R[F(\xi_2)]-c_pF(\xi_2)}\Phi(\xi_1)\,d\xi_1-\right.$$

$$\left.-\frac{\rho_0}{\alpha_2}\int\limits_{\dot{R}_0}^{r+c_pt}d\xi_3\int\limits_{\dot{R}_0}^{\xi_3}\frac{\dot{R}\,[F(\xi_2)]\,\ddot{R}\,[F(\xi_2)]\,R\,[F(\xi_2)]}{\Delta_2\,[F(\xi_2)]}\,d\xi_2+m_1c_pt+n_1\right\},$$

where

$$m_1=-\frac{R_0^2}{c_p}\{[\dot{R}(0)+c_p]\,\dot{\varepsilon}^*(0)+\ddot{R}(0)\,\varepsilon^*(0)\};\quad n_1=-R_0^2\dot{R}(0)\,\varepsilon^*(0);$$

$$\Phi(\xi_1)=-\frac{\dot{R}^3(F(\xi_1))\,\Delta_1(F(\xi_1))}{2c_pR(\dot{R}-c_p)}\left[1+\frac{R\ddot{R}}{\dot{R}^2}\left(6+\frac{R\dddot{R}}{\dot{R}\ddot{R}}\right)\right]-$$

$$-\frac{\frac{\rho_0}{\alpha_2}\dot{R}^3\ddot{R}}{2c_p(\dot{R}-c_p)\,\Delta_2(F(\xi_1))}\left[2+\frac{R\ddot{R}}{\dot{R}^2}-\frac{c_p(\dot{R}+c_p)}{\dot{R}^2}\right]-$$

$$-\frac{\frac{\rho_0}{\alpha_2}R\dot{R}\ddot{R}^2}{2c_p(\dot{R}-c_p)\,\Delta_2^3(F(\xi_1))}\left\{\Delta_2^2(F(\xi_1))\left[2+\frac{(2\dot{R}+c_p)}{\dot{R}}\left(1+2\frac{\dot{R}^2}{R\ddot{R}}+\frac{\dot{R}\dddot{R}}{\ddot{R}^2}\right)\right]+\frac{\rho_0}{\alpha_2}\dot{R}(2\dot{R}+c_p)\right\}.$$

The derivation of an equation for the load $p_0(t)$ in the case of a spherical wave taking (1.13) into account is similar to the case of a plane wave

$$p_0(t)=p^*(t)+\rho_0c_p\int\limits_{R_0}^{R(t)}\frac{1}{r}\left\{-\int\limits_{\dot{R}_0}^{r-c_pt}\Phi(\xi_1)\,d\xi_1-\int\limits_{\dot{R}_0}^{R[F(r+c_pt)]-c_pF(r+c_pt)}\Phi(\xi_1)\,d\xi_1-\frac{\rho_0\dot{R}\,[F(r+c_pt)]\,\ddot{R}R}{\alpha_2\Delta_2\,[F(r+c_pt)]}\right\}dr- \tag{1.14}$$

$$-\rho_0c_p\int\limits_{R_0}^{R(t)}\frac{1}{r^2}\left\{-\int\limits_{\dot{R}_0}^{r-c_pt}d\xi_2\int\limits_{\dot{R}_0}^{\xi_2}\Phi(\xi_1)\,d\xi_1-\int\limits_{\dot{R}_0}^{r+c_pt}d\xi_2\int\limits_{\dot{R}_0}^{R[F(\xi_2)]-c_pF(\xi_2)}\Phi(\xi_1)\,d\xi_1-\frac{\rho_0}{\alpha_2}\int\limits_{\dot{R}_0}^{r+c_pt}\frac{\dot{R}\,[F(\xi_2)]\,\ddot{R}R}{\Delta_2\,[F(\xi_2)]}\,d\xi_2+m_1\right\}dr.$$

As noted above, Eq. (1.14) holds when $p_0(t)\geqslant 0$, and if $p_0(t)$ vanishes in a finite time interval $t = t_0$ (see Fig. 1), it is necessary to carry out additional investigations by dividing the region into n parts, as was done in the plane case.

To determine the shape of the shock wave R(t) in region 3, unlike the plane case we have a nonlinear differential equation of the form

$$\frac{d\ddot{R}(t)}{dt}=\frac{1}{F(\dot{R})}\left\{-\frac{2c_p(a_1-c_p)\,\psi_1'''\,[R_1+a_1(t-t_1)-c_pt]}{[R_1+a_1(t-t_1)]}-\right. \tag{1.15}$$

$$-\frac{\frac{\rho_0}{\alpha_2}a_1^2\dot{R}\ddot{R}\left[\left(1-\frac{c_p^2}{a_1^2}\right)+\left(1+\frac{c_p}{a_1}\right)+4\frac{\dot{R}(t)}{a_1}\right]}{[R_1+a_1(t-t_1)]\,\Delta_2(t)}-$$

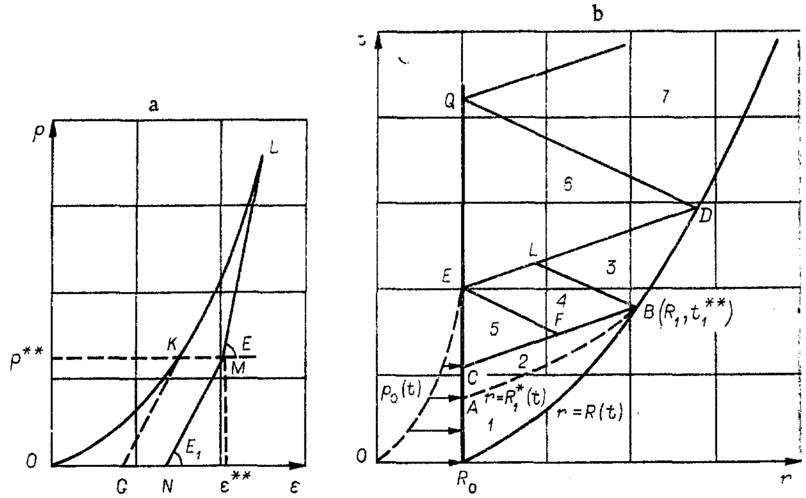

Fig. 2

$$-\frac{\frac{\rho_0}{\alpha_2} a_1 \ddot{R}^2\left[\left(1+\frac{c_p}{a_1}\right)-3\frac{\dot{R}(t)}{a_1}\right]}{\Delta_2(t)} - \frac{\left(\frac{\rho_0}{\alpha_2}\dot{R}\ddot{R}\right)^2 a_1\left[\left(1+\frac{c_p}{a_1}\right)+\frac{\dot{R}(t)}{a_1}\right]}{\Delta_2^3(t)} -$$

$$-\frac{2a_1\Delta_2(t)}{[R_1+a_1(t-t_1)]}\left[2\ddot{R}(t)+\frac{a_1\dot{R}(t)}{R_1+a_1(t-t_1)}\right],$$

$$\text{where}\quad F(\dot{R})=\frac{\frac{\rho_0}{\alpha_2}\dot{R}(t)\,[\dot{R}(t)+(c_p+a_1)]}{\Delta_2(t)}+\Delta_2(t);$$

$\psi_1(t)$ is a function known from the solution of the problem in region 3, and the prime denotes the derivative with respect to the argument. Equation (1.15) can be solved numerically by the Runge–Kutta method [6], as a Cauchy problem with initial data $\dot{R}=\dot{R}(t_1)$, $\ddot{R}=\ddot{R}(t_1)$ with $t = t_1$. The procedure for obtaining the formula for the pressure in regions 2 and 3 is based on integration of the equation of motion of the medium with appropriate limits of integration taking into account the boundary conditions of the problem.

2. Propagation of Waves in a Medium with Discontinuous Unloading. If the diagram of the state of the medium (Fig. 2a) for unloading has a broken line, consisting of two straight lines, the results obtained in section 1 hold as long as $p(r, t) \geqslant p^{**}$ and $\varepsilon \geqslant \varepsilon^{**}$. Hence using the results obtained above in the physical (r, t) plane we initially determine the surface on which $p = p^{**}$, $\varepsilon = \varepsilon^{**}$ and find the velocity distribution on it. Calculations show that the pressure on the shock-wave front decays more slowly than in the cavity. Hence, the pressure isobar is elongated towards the spatial coordinate r (Fig. 2b). Further when $p < p^{**}$ the medium becomes less rigid and possesses a Young's modulus E_1 ($E_1 < E$), and it is then necessary to solve the problem of the propagation of plane and spherical waves for region 2, bounded by the surface AB, the characteristic of the positive direction BC, and the boundary of the layer AC (see Fig. 2b).

We will solve the plane problem with the following boundary conditions:

$$\left.\begin{aligned} u(r,t)=u^{**}(t),\quad \frac{\partial u}{\partial t}(r,t)=\dot{u}^{**}(t)\\ p=p^{**}=\text{const},\quad \varepsilon=\varepsilon^{**}=\text{const}\end{aligned}\right\}\ \text{for}\quad r=R_1^*(t). \tag{2.1}$$

The equation of state of the medium in this case has the form

$$p(r,t)=p^{**}+E_1(\varepsilon-\varepsilon^{**}), \tag{2.2}$$

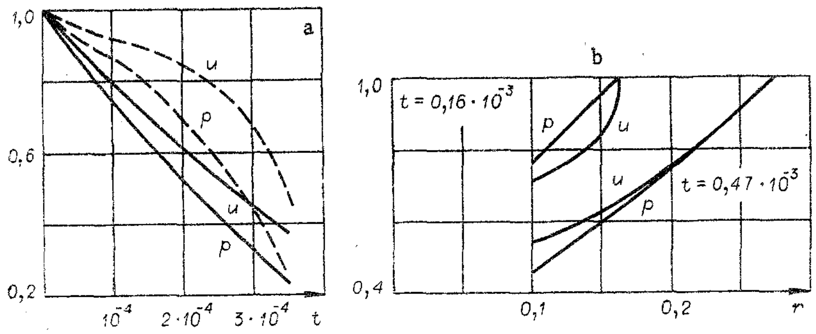

Fig. 3

where $E_1 = \rho_0 c_{p1}^2$; p^{**}, ε^{**} are specified quantities found from the $p \sim \varepsilon$ diagram. Then (1.4), taking (2.1) and (2.2) into account, has the solution

$$u(r,\ t) = u^{**}\left(R_0,\ t_0^{**}\right) - \frac{1}{2c_{p1}}\left\{\int\limits_{z_{30}}^{r-c_{p1}t} \dot{u}^{**}\left[F_3(z_3)\right] dz_3 - \int\limits_{z_{40}}^{r+c_{p1}t} \dot{u}^{**}\left[F_4(z_4)\right] dz_4\right\},$$

where $z_{i0} = R_0 \mp c_{p1}t_0^{**}$; $F_i(z_i)$ $(i = 3,\ 4)$ is the root of the equation $R_1^*(t) \mp c_{p1}t = z_i$ for the time t. In this case using (2.1) we obtain from (1.1)

$$p_0(t) = p^{**} - \frac{\rho_0}{2}\int\limits_{R_1^*(t)}^{R_0}\left\{\dot{u}^{**}\left[F_3(r - c_{p1}t)\right] + \dot{u}^{**}\left[F_4(r + c_{p1}t)\right]\right\} dr. \tag{2.3}$$

Similar investigations in the case of a spherical wave enable us to represent the solution of the problem in region 2 in the form

$$u(r,\ t) = \frac{1}{r}\left\{\int\limits_{z_{30}}^{r-c_{p1}t} d\xi \int\limits_{z_{30}}^{\xi} K_4(\xi_1)\, d\xi_1 + \int\limits_{z_{40}}^{r+c_{p1}t} d\xi \int\limits_{z_{40}}^{\xi} K_5(\xi_1)\, d\xi_1\right\} - \tag{2.4}$$

$$-\frac{1}{r^2}\left\{\int\limits_{z_{30}}^{r-c_{p1}t} d\xi \int\limits_{z_{30}}^{\xi} d\xi_2 \int\limits_{z_{30}}^{\xi_2} K_4(\xi_1)\, d\xi_1 + \int\limits_{z_{40}}^{r+c_{p1}t} d\xi \int\limits_{z_{40}}^{\xi} d\xi_2 \int\limits_{z_{40}}^{\xi_2} K_5(\xi_1)\, d\xi_1 + m_5 c_{p1}^2 t^2 + n_5 c_{p1} r t + m_6 c_{p1} t + n_6\right\};$$

$$p_0(t) = p^{**} - \rho_0 c_{p1}\int\limits_{R_1^*(t)}^{R_0}\left\{\frac{1}{r}\left[-\int\limits_{z_{30}}^{r-c_{p1}t} K_4(\xi_1)\, d\xi_1 + \int\limits_{z_{40}}^{r+c_{p1}t} K_5(\xi_1)\, d\xi_1\right] - \frac{1}{r^2}\left[-\int\limits_{z_{30}}^{\xi_2} K_4(\xi_1)\, d\xi_1 + \right.\right. \tag{2.5}$$

$$\left.\left. + \int\limits_{z_{40}}^{r+c_{p1}t} d\xi_2 \int\limits_{z_{40}}^{\xi_2} K_5(\xi_1)\, d\xi_1 + 2m_5 c_{p1} t + n_5 r + m_6\right]\right\} dr,$$

$$K_4(\xi) = \frac{1}{\left\{\dfrac{\dot{R}_1^*\left[F_3(\xi)\right]}{c_{p1}} - 1\right\}}\left\{\frac{\dot{u}^{**}\left[F_3(\xi)\right]}{c_{p1}} - \frac{u^{**}\left[F_3(\xi)\right]}{R_1^*\left[F_3(\xi)\right]} - \frac{2\dot{R}_1^*\left[F_3(\xi)\right]\dot{u}^{**}\left[F_3(\xi)\right] + R_1^*\left[F_3(\xi)\right]\ddot{u}^{**}\left[F_3(\xi)\right]}{2c_{p1}^2}\right\},$$

$$K_5(\xi) = \frac{\dot{u}^{**}\left[F_4(\xi)\right]}{c_{p1}} + \frac{1}{\left\{1 + \dfrac{\dot{R}_1^*\left[F_4(\xi)\right]}{c_{p1}}\right\}}\left\{\frac{u^{**}\left[F_4(\xi)\right]}{R_1^*\left[F_4(\xi)\right]} + \frac{R_1^*\left[F_4(\xi)\right]\ddot{u}^{**}\left[F_4(\xi)\right]}{2c_{p1}^2}\right\},$$

where m_5, n_5, m_6, n_6 are constant coefficients calculated from the first two conditions of (2.1) when $R_1^*(t)$ approaches the points $A(R_0,\ t_0^{**})$ and $B(R_1,\ T_1^{**})$ (see Fig. 2b).

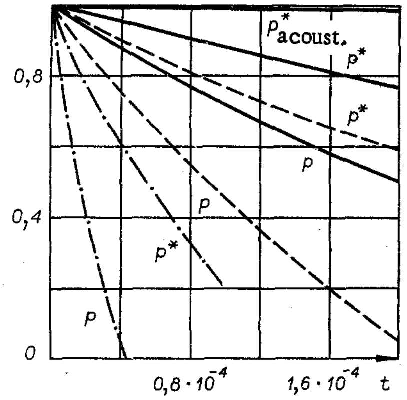

Fig. 4

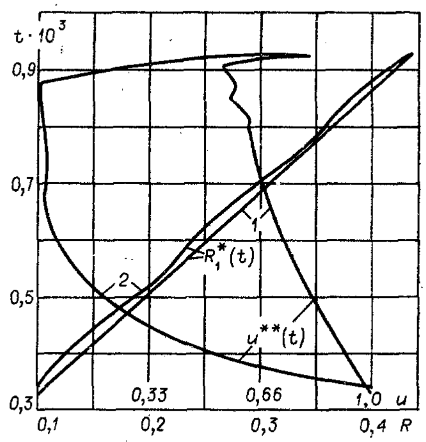

Fig. 5

A comparative investigation shows that the horizontal line MK in the diagram (see Fig. 2a) corresponds to the surface AB (see Fig. 2b) in the physical (r, t) plane. It may happen that the load defined by (2.3) or (2.5) vanishes at the intermediate point E at the boundary of the layer. Regions 3, 4, and 5 then occur bounded respectively by the characteristics of the positive and negative directions, the boundary line CE and the part of the shock wave BD. The method of solving the problem in region 3 was described above, while in regions 4 and 5 we have a Gurs problem. Further, in regions 6 and 7 (see Fig. 2b) the problem is solved in the same way as in regions 2 and 3 of section 1 (see Fig. 1). It is of interest to investigate the effect of the duration and profile of the load on the shock-wave processes.

3. Results of the Calculations. Some results of the calculations for a medium with linear unloading with the initial parameters [7]

$$\alpha_1 = 12.127\cdot 10^2\ \mathrm{kg/cm^2},\ \alpha_2 = 58.73\cdot 10^3\ \mathrm{kg/cm^2},\quad E = 14\cdot 10^3\ \mathrm{kg/cm^2},\ R_1 = 391\ \mathrm{m/sec}; \tag{3.1}$$

$$\alpha_1 = 18\cdot 10^2\ \mathrm{kg/cm^2},\ \alpha_2 = 82\cdot 10^3\ \mathrm{kg/cm^2},\ E = 18\cdot 10^3\ \mathrm{kg/cm^2},\quad R_1 = 440\ \mathrm{m/sec}\,(p_0 = 105\ \mathrm{kg/cm^2},\ \rho_0 = 200\ \mathrm{kg\cdot sec^2/m^4}, R_2 = 2R_1\cdot 10^2\ \mathrm{m/sec^2}) \tag{3.2}$$

for the case when the shape of the surface of the front is specified in the form of a second-degree polynomial

$$R(t) = R_0 + R_1 t - R_2\frac{t^2}{2},$$

where $\dot{R}(t) \geqslant 0$, are given in Figs. 3-5 in dimensionless form with respect to the maximum value of the pressure, velocity, and the units of length and time. In Fig. 3a we give graphs of the change in the load $p_0(t)$ and the mass velocity of the medium u(t) on the boundary of a plane and spherical layer (the broken lines) and on the front R(t) as a function of time. Hence it can be seen that to maintain the same pressure at corresponding points of the plane and spherical front it is necessary to apply a larger load to a spherical cavity compared with a plane one. This is a consequence of the inverse formulation of the problem, since in the direct formulation (if the load is given) the pressure on the spherical front falls more rapidly than on the plane front. In this case the decay of the pressure (velocity) on the wavefront occurs more slowly than on the boundary of the layer.

In Fig. 3b we show the change in p(r, t) and u(r, t) as a function of the spatial coordinate r at a fixed instant of time t. Notice that the pressure varies linearly with r, whereas the velocity varies mainly nonlinearly. In order to investigate the dependence of the load $p_0(t)$ and the pressure $p^*(t)$ on the shape of the shock-wave front we show curves in Fig. 4 of the change in $p_0(t)$ and $p^*(t)$ as a function of t for case (3.2) for $R_2 = 2R_1\cdot 10^2$, $4R_1\cdot 10^2$, and $2R_1\cdot 10^3$; these are shown by the continuous, broken, and dash-dot lines respectively. These curves indicate that in the plane problem when $R_2 = 4R_1\cdot 10^2$ and $R_2 = 2R_1\cdot 10^3$ the pressure $p^*(t)$ and the load $p_0(t)$ fall nonlinearly as p increases. In Fig. 5 we show the surface of constant pressure p^{**} = const and a curve of the velocity distribution $u^{**}(t)$

on it as a function of t, which serves as the boundary condition when investigating the propagation of a plastic wave in a nonlinearly compressed medium in the following regions (curve 2 refers to a spherical wave).

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